PART VI

Macroeconomic Implications and the Political Economy of Corporate Finance

13

Credit Rationing and Economic Activity

13.1 Introduction

In the first issue of *Econometrica* (1933), Irving Fisher stressed the key role of credit constraints in amplifying and protracting the ongoing recession. The combination of nonindexed debt contracts and deflation, he argued, redistributed wealth from borrowers to creditors; furthermore, the reduction in the firms' cash flows and the fall in collateral values increased leverage and reduced investment, thereby exacerbating the recession. Fisher's prescient concern about what are now called balanced sheet effects has received substantial empirical, microeconomic, and macroeconomic support since his time. For example, numerous contributions have established links between high leverage ratios, falling asset prices, and low investment and economic activity (see, for example, King (1994) and Bernanke et al. (1999) for an overview).

While this "balance-sheet channel" refers to the influence of firms' balance sheets on their investment and production, the "lending channel," in contrast, focuses on the impact of the strength of financial intermediaries' balance sheets on firms' activity. At the microeconomic level, firms with weak balance sheets (often, small firms) depend on monitoring and certification by financial intermediaries (banks and insurance companies) to secure access to funds. They are thus hurt when banks' and insurance companies' real or regulatory solvency declines. Similarly, the market for initial public offerings of technology companies closed after the Internet and communications company stocks collapsed in 2000; venture capitalists (the intermediaries monitoring and certifying start-ups) were then deprived of an exit strategy and consequently lacked funds to finance new start-ups. It took a couple of years for technology finance to start recovering.

At the macroeconomic level, economists, starting with Bernanke (1983), have documented the contractionary impact on loans of a tight monetary policy (an increase in the federal funds rate) and a concomitant increase in commercial paper issues (showing that the contraction is related to a reduction in loan supply rather than to a decrease in loan demand). Related observations point at the negative impact of bank panics on macroeconomic activity (Friedman and Schwartz 1963) and at the incidence of the tax associated with bank reserves requirements on bank borrowers rather than on depositors (Fama 1985; James 1987).

This chapter provides a theoretical analysis of the balance-sheet channel (Section 13.2) and of the lending channel (Section 13.3). Sections 13.4 and 13.5 study the dynamic linkages in infinite-horizon models with successive generations. Section 13.4 focuses on dynamic complementarities due to net worth accumulation and shows how short-term balance sheet effects can have a long-term impact on the welfare of either individual families or whole countries. Section 13.5, in contrast, looks at dynamic lending substitutabilities and investigates the negative effect of today's investment on future prices and thereby on future investment.

13.2 Capital Squeezes and Economic Activity: The Balance-Sheet Channel

This section analyses the impact of interest rates on economic activity when the corporate sector faces credit constraints. It revisits the basic moral-hazard and adverse-selection models of Chapters 3 and 6, and generalizes them by endogenizing the rate of interest. Taking the interest rate as exogenous (and normalizing it to 0 without loss of generality) was fine until now, since we were focusing on the institutions of corporate finance. Moving to a macroeconomic framework, however, requires endogenizing the rate of interest, unless the savings function is perfectly elastic at some fixed interest rate, such as the interest rate on the world financial markets.

Namely, letting r denote the (real) rate of interest, we posit a savings function $S(r)$, increasing in r. This function can be derived from investors' preferences: let date 0 denote the date at which they lend and date 1 the date at which their claims on firms pay off, with associated consumptions c_0 and c_1; and let investors' preferences be given by

$$U(c_0, c_1) = u(c_0) + c_1,$$

where $u(\cdot)$ is increasing and concave. This formulation is handy since it preserves risk neutrality with respect to returns (and thus the concomitant simplicity) while making the saving function imperfectly elastic. The saving function is then obtained from

$$\max_{\{c_0,c_1\}} \{u(c_0) + c_1\}$$

s.t.

$$c_0 + \frac{c_1}{1+r} = y,$$

where y denotes income. This program is equivalent to[1]

$$\max_{\{c_0\}} \{u(c_0) + (1+r)(y - c_0)\},$$

yielding

$$u'(c_0(r)) = 1 + r.$$

Because u is concave ($u'' < 0$), date-0 consumption decreases with the rate of interest. Savings, $S(r) = y - c_0(r)$, in contrast, increase with the rate of interest.

The extreme case of a perfectly elastic savings function, in which the interest rate is fixed at some exogenous level and is given by a "storage technology," or some "international rate," or else by fully linear investors' preferences $(c_0 + c_1/(1 + r))$, provides a special case of savings function relative to this more general environment.

The theme of this section, the aim of which is primarily to introduce basic material, is that an increase in the rate of interest has a negative impact on investment. It is not very surprising, you might say, that when the price of a factor of production (here capital) increases, the use made of this factor of production decreases. It holds whether or not firms face financial constraints. The interesting insight is that interest rates may have very sharp effects in a corporate finance world, as credit constraints exacerbate their impact. Indeed, a small increase in the interest rate may trigger a complete collapse of lending and a discontinuous reduction in welfare.

13.2.1 Moral Hazard

Let us first revisit the basic, fixed-investment model of Section 3.2.

Consider a set of risk-neutral entrepreneurs, technically a continuum of mass 1 of them. Each has

- a project requiring fixed investment I, and owns assets or net worth A;
- a utility function of consumptions c_0 and c_1 at dates 0 and 1 equal to $U(c_0, c_1) = c_0 + c_1$; entrepreneurs are protected by limited liability (in particular, $c_1 \geqslant 0$).

The entrepreneurs' particular utility function is in no way crucial. What is required more generally is that the entrepreneurs not be more impatient than the savers,[2] because otherwise the direction of lending might be reversed, with limited interest for our purpose. In this spirit, we will assume that the equilibrium rate of interest is positive ($r > 0$).

If undertaken, the project either succeeds, that is, yields verifiable income $R > 0$, or fails and yields no income. The probability of success, p, depends on the entrepreneur's behavior: it is equal to p_{H} if the entrepreneur works and $p_{\mathrm{L}} = p_{\mathrm{H}} - \Delta p$ if she shirks. Shirking yields a private benefit $B > 0$ to the entrepreneur (this private benefit is counted as part of c_1).

We allow for one dimension of heterogeneity: entrepreneurs differ in their assets A. Namely, A, which recall is an index of a firm's strength of

1. We assume an interior equilibrium. This is indeed the case if $u'(0) > 1 + r > u'(y)$.

2. So, for example, in the extreme case in which the savings function is perfectly elastic at some interest rate r, entrepreneurs could have preferences

$$c_0 + \frac{c_1}{1+r}$$

without any change in the analysis.

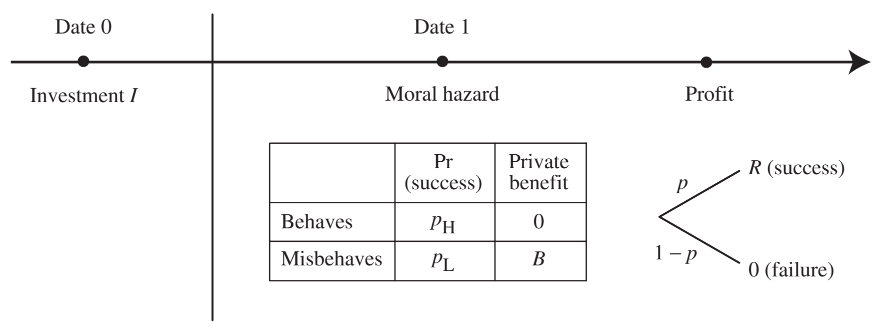


Figure 13.1

balance sheet, is distributed in the population of entrepreneurs according to the continuous cumulative distribution function $G(A)$ with support $[\underline{A}, \bar{A}]$ and density $g(A)$. The upper bound $\bar{A}$ in principle can exceed I; firms with assets $A \geqslant I$ do not need to borrow in order to invest and are therefore net savers; needless to say, those "borrowers" do not preoccupy us. For simplicity, we will therefore assume that $\bar{A} \leqslant I$.

The timing is the familiar one (see Figure 13.1), except that we have now got to be careful about dates since the interest rate may now differ from 0.[3]

We assume that the project has positive NPV if and only if the entrepreneur behaves. That is, in the relevant range for interest rates,

$$p_H R > (1 + r)I > p_L R + B.$$

To solve for the macroeconomic equilibrium, we proceed as in Section 3.2. Conditional on the entrepreneur receiving funding, the optimal contract allocates the profit in the case of success between borrower (R_b) and lenders (R_l),

$$R = R_b + R_l,$$

and gives 0 to both in the case of failure (recall that the entrepreneur is risk neutral and therefore must receive the harshest punishment in the case of failure, namely, 0 under limited liability). The incentive compatibility constraint is

$$(\Delta p) R_b \geqslant B,$$

and so the maximum expected income that can be pledged to investors without destroying incentives—the pledgeable income—is equal to

$$p_H\left(R - \frac{B}{\Delta p}\right).$$

A necessary and sufficient condition[4] for an entrepreneur with assets A to receive financing is

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant (1 + r)(I - A).$$

Let $A^*(r)$ (an increasing function) be the smallest level of cash on hand A that enables funding:

$$p_H\left(R - \frac{B}{\Delta p}\right) = (1 + r)[I - A^*(r)].$$

The financial market clears when corporate net investment, $\mathcal{I}(r)$, is equal to investors' savings; $\mathcal{I}(r)$ is given by

$$\begin{aligned}\mathcal{I}(r) &\equiv \int_{A^*(r)}^{\bar{A}} (I - A) g(A)\,\mathrm{d}A - \int_{\underline{A}}^{A^*(r)} A g(A)\,\mathrm{d}A \\ &= (1 - G(A^*(r)))I - A^{\mathrm{e}},\end{aligned}$$

where

$$A^{\mathrm{e}} \equiv \int_{\underline{A}}^{\bar{A}} A g(A)\,\mathrm{d}A$$

is the average entrepreneur wealth. Market clearing means that

$$\mathcal{I}(r) = S(r).$$

This equilibrium is depicted by point a in Figure 13.2.

The comparative statics are straightforward. Consider, first, an exogenous reduction in the savings rate. That is, the savings curve moves up in Figure 13.2. Unsurprisingly, the equilibrium shifts to

3. Locating the moral-hazard stage at date 1 rather than date 0 is just an accounting convention, and has no impact on the results.

4. This condition is necessary since the NPV is negative and so someone has to lose if the contract induces shirking. It is easy to see that it is also sufficient. See Section 3.2 for more details.

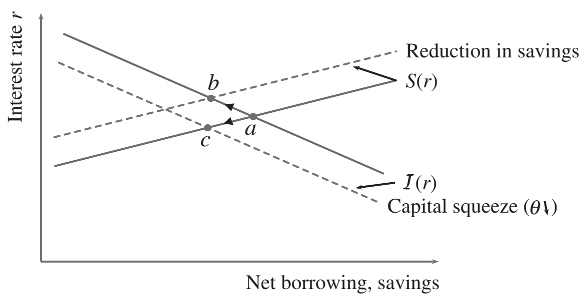


Figure 13.2

point b, with lower investment and an increase in the interest rate.

Let us next look at a deterioration in the firms' balance sheets. The proper way to formalize an overall change in the distribution of the balance sheets is to assume that the distribution of assets A is indexed by a parameter θ, $G(A \mid \theta)$, and that an increase in θ corresponds to an improvement of the distribution in the sense of first-order stochastic dominance:

$$G_\theta(A \mid \theta) < 0 \quad \text{for } \underline{A} < A < \bar{A},$$

where a subscript here denotes a partial derivative ($\partial G/\partial\theta \equiv G_\theta$). Intuitively, when θ increases, the distribution puts more weight on the upper tail and less on the lower tail.[5]

Net borrowing $\mathcal{I}(r, \theta)$ is affected by a capital squeeze (θ decreases) in the following way:

$$\mathcal{I}_\theta = -G_\theta(A^*(r))I - \frac{\mathrm{d}A^{\mathrm{e}}}{\mathrm{d}\theta}.$$

Thus a capital squeeze has two effects:

Eviction (indirect effect). The number of firms that are unable to raise funds because of the weakness of their balance sheet, $G(A^*(r) \mid \theta)$, increases as θ decreases. The firms that are evicted from the pool of borrowers are the marginal firms, which borrowed $I - A^*(r)$. The decrease in the demand for funds corresponds to the first term in the expression of $\mathcal{I}_\theta$;

Greater needs (direct effect). Because

$$A^{\mathrm{e}}(\theta) = \int_{\underline{A}}^{\bar{A}} A \,\mathrm{d}G(A \mid \theta) = \bar{A} - \int_{\underline{A}}^{\bar{A}} G(A \mid \theta)\,\mathrm{d}A$$

(after an integration by parts), $\mathrm{d}A^{\mathrm{e}}(\theta)/\mathrm{d}\theta > 0$. Hence, a capital squeeze reduces the entrepreneurs' average net worth.

Thus, the $\mathcal{I}(\cdot)$ curve may shift outward or (as depicted in Figure 13.2) inward. (As we will observe, this indeterminacy is removed in the variable-investment version.) For example, if the eviction effect dominates, a capital squeeze moves the equilibrium in Figure 13.2 from point a to point c, with a lower level of net borrowing and a lower interest rate.

The investment level, equal to I times the fraction of firms that have access to funding,

$$1 - G(A^*(r) \mid \theta),$$

in contrast is unambiguously reduced by a capital squeeze.[6]

Remark (on redistribution). The literature has emphasized that wealth redistribution has an ambiguous impact on efficiency (leaving aside redistributive aspects of course). While this point has often been made in more sophisticated growth models (e.g., of the type reviewed in Sections 13.4 and 13.5), the basic idea can be conveyed in the static version. A redistribution of wealth, namely, a change in the distribution of wealth levels A keeping total entrepreneur wealth, A^{e}, constant,[7] affects in an ambiguous way the number of firms that make it to the borrowing threshold. For example, suppose that there are two

5. See, for example, Mas Colell et al.'s (1995) textbook for an exposition of first-order stochastic dominance.

Note that, because $G(\bar{A} \mid \theta) = 1$ and $G(\underline{A} \mid \theta) = 0$ for all θ,

$$G_\theta(\bar{A} \mid \theta) = G_\theta(\underline{A} \mid \theta) = 0.$$

A special case is that in which θ is a uniform shift in A (each A becomes $A+\theta$): $G(A \mid \theta) = H(A-\theta)$, where H is a cumulative distribution function. (This case involves a "moving support." And so if $\theta \in [\underline{\theta}, \bar{\theta}]$, the inequality $G_\theta \leqslant 0$ is weak over two ranges in $[\underline{A} + \underline{\theta}, \bar{A} + \bar{\theta}]$.)

6. Its total derivative with respect to θ is (I times)

$$-G_\theta(A^*(r) \mid \theta) - g(A^*(r) \mid \theta)\frac{\mathrm{d}A^*}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}\theta}.$$

By definition, $G_\theta < 0$. Also, $\mathrm{d}A^*/\mathrm{d}r > 0$ (a higher interest rate leads to the eviction of marginal firms). Finally,

$$S'(r)\,\mathrm{d}r = \mathcal{I}_r\,\mathrm{d}r + \mathcal{I}_\theta\,\mathrm{d}\theta.$$

And so

$$\frac{\mathrm{d}}{\mathrm{d}\theta}[1 - G(A^*(r) \mid \theta)] = \left(-G_\theta S' + g\frac{\mathrm{d}A^*}{\mathrm{d}r}\frac{\mathrm{d}A^{\mathrm{e}}}{\mathrm{d}\theta}\right)\Big/\left(S' + gI\frac{\mathrm{d}A^*}{\mathrm{d}r}\right) > 0.$$

7. The literature often considers a specific form of wealth redistribution, namely, a mean-preserving decrease in risk for the distribution G (so the parameter θ is now a parameter of second-order stochastic dominance rather than one of first-order stochastic dominance).

For a mean-preserving spread,

$$\frac{\mathrm{d}A^{\mathrm{e}}}{\mathrm{d}\theta} = \int_{\underline{A}}^{\bar{A}} A\,\mathrm{d}G_\theta(A \mid \theta) = 0 \quad \text{and} \quad \int_{\underline{A}}^{A} G_\theta(A \mid \theta) \geqslant 0 \quad \text{for all } A.$$

But $\mathcal{I}_\theta = -G_\theta(A \mid \theta)I$ can *a priori* have any sign.

levels of wealth, A_L and A_H, in the population and that the savings function is perfectly elastic, so that the rate of interest r and the threshold $A^*(r)$ are exogenously determined. A wealth-redistribution policy brings these to A'_L and A'_H, where $A_L < A'_L \leqslant A'_H < A_H$. If the threshold lies between A'_H and A_H, then the wealth redistribution eliminates the entrepreneurial class and reduces efficiency. If it lies between A_L and A'_L, then wealth redistribution allows everyone to be an entrepreneur and increases efficiency.[8]

Variable-investment variant. The same exercise can be performed for a variable investment scale (Section 3.4). The entrepreneur selects a scale $I \in [0, \infty)$. Profit in the case of success (RI) is proportional to investment; there is still no profit in the case of failure. Misbehavior, which, as in the fixed-investment model, reduces the probability of success from p_H to p_L, yields private benefit, BI, proportional to investment, to the entrepreneur. We assume that, in the relevant range of interest rates, the following inequalities, where the magnitudes are expressed per unit of investment, hold:

$$p_H R > 1 + r > \max\left\{p_L R + B, p_H\left(R - \frac{B}{\Delta p}\right)\right\}.$$

The first inequality says that investing is a positive-NPV proposition if incentives are in place. The second inequality says, first, that the NPV is negative if the entrepreneur is induced to misbehave $(1 + r > p_L R + B)$, and, second, that the pledgeable income per unit of investment does not cover interest and principal on the loan $(1 + r > p_H(R - B/\Delta p))$—this assumption, as in Section 3.4, will guarantee that the optimal investment is finite in this constant-returns-to-scale model.

Letting R_b denote the entrepreneur's reward in the case of success (it is 0 in the case of failure), the incentive compatibility constraint is

$$(\Delta p) R_b \geqslant BI,$$

yielding pledgeable income

$$p_H RI - p_H\Big\{\min_{\{R_b \geqslant BI/\Delta p\}} R_b\Big\} \equiv p_H\left(R - \frac{B}{\Delta p}\right) I,$$

and so the investors' breakeven condition (which, due to the competitiveness of the capital market, holds with equality) is

$$p_H\left(R - \frac{B}{\Delta p}\right) I = (1 + r)(I - A).$$

As in Section 3.4, the investment scale is a multiplier of assets:

$$I = \frac{A}{1 - p_H(R - B/\Delta p)/(1 + r)}.$$

Note that

- an increase in the rate of interest reduces the scale of investment,
- all firms are identical up to their scale, and so the distribution of assets among entrepreneurs is irrelevant here (unlike in the fixed-investment case) for a given level of total assets $A^e = \int_{\underline{A}}^{\bar{A}} A g(A)\, dA$.

Indeed, net borrowing for a distribution indexed by parameter θ is

$$\begin{aligned} \mathcal{I}(r, \theta) &\equiv \int_{\underline{A}}^{\bar{A}} (I - A) g(A \mid \theta)\, dA \\ &= \frac{p_H(R - B/\Delta p)}{(1 + r) - p_H(R - B/\Delta p)} A^e(\theta), \end{aligned}$$

where

$$A^e(\theta) \equiv \int_{\underline{A}}^{\bar{A}} A g(A \mid \theta)\, dA.$$

As in the fixed-investment version, let us assume that θ is a parameter of first-order stochastic dominance: $G_\theta < 0$. Integrating by parts, and using $G_\theta(\bar{A} \mid \theta) = G_\theta(\underline{A} \mid \theta) = 0$,[9]

$$\frac{dA^e(\theta)}{d\theta} = -\int_{\underline{A}}^{\bar{A}} G_\theta(A \mid \theta)\, dA > 0.$$

Hence, in the variable-investment variant, the investment is scaled down when a firm has lower assets and $\mathcal{I}$ unambiguously shifts inward with a capital squeeze (θ decreases), as depicted in Figure 13.2. Furthermore, as in the case of a fixed investment size, a reduction in savings leads to a higher rate of interest and a smaller investment.[10]

13.2.2 Adverse Selection

As we studied in Chapter 6, adverse selection (the presence of entrepreneurial private information at

8. This mechanism is not the only cause of ambiguity. See Aghion and Bolton (1997) for a more complete discussion.

9. Since $G(\bar{A} \mid \theta) = 1$ and $G(\underline{A} \mid \theta) = 0$ for all θ.

10. Note also that a mean-preserving spread in the distribution of net worths has no impact on investment.

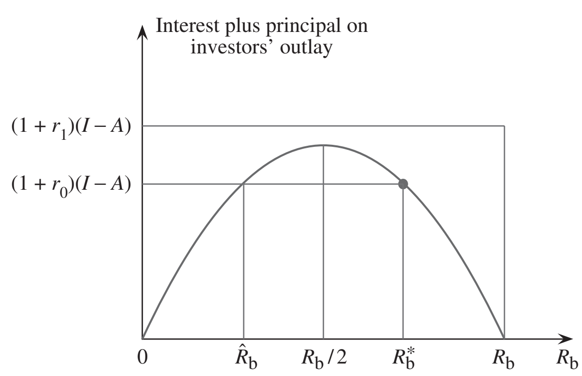


Figure 13.3

the initial financing stage) is another factor conducive to credit rationing. Under adverse selection, the impact of an interest rate increase may be dramatic, as we will shortly see. The increased debt burden may create a serious deterioration of the pool of loan applicants.[11] Conversely, a small improvement in lending conditions may have a substantial impact on economic activity; along these lines, Mankiw (1986) argues that small government interventions (e.g., subsidized loans to students, farmers, and homeowners) can make a big difference.

This section (building on Chapter 6) offers two illustrations of the potentially strong impact of interest rates on activity in the presence of adverse selection. Both illustrations use the fixed-investment version of the model.

(a) *Impact of factor price on behavior: asymmetric information on private benefits.* Let us assume that all borrowers have the same fixed-investment technology and the same level of assets $A < I$. The source of heterogeneity is the level of private benefit B obtained by the entrepreneur when misbehaving. The latter is distributed in the population of entrepreneurs on the interval $[0, \bar{B}]$ according to cumulative distribution function $H(B)$ (with $H(0) = 0$, $H(\bar{B}) = 1$).

Investors would want to screen out "bad types," namely, those with high private benefits from misbehaving. Unfortunately (and as was observed in Exercise 6.1), such types cannot be screened out since their surplus from the lending relationship is at least equal to that of entrepreneurs with a lower private benefit.

11. See, for example, Jaffee and Russell (1976), Stiglitz and Weiss (1981), and Mankiw (1986), who, in the tradition of Akerlof (1970), demonstrate the dramatic impact of adverse selection in the market for loans.

Suppose that investors are willing to finance the project of a representative entrepreneur ("representative" from the point of view of investors, as entrepreneurs all look alike). The entrepreneur contributes A and the investors $I - A$. They share the profit in the case of success in proportions R_b and $R - R_b$, respectively. Provided that $\bar{B}$ is sufficiently large, the entrepreneur behaves if $B < B^*(R_b)$ and misbehaves if $B > B^*(R_b)$, where $B^*(R_b)$ is given by

$$(\Delta p)R_b = B^*(R_b). \tag{13.1}$$

The investors' breakeven condition is then

$$\hat{p}(R_b)[R - R_b] = (1+r)(I-A), \tag{13.2}$$

where

$$\hat{p}(R_b) \equiv p_H H(B^*(R_b)) + p_L[1 - H(B^*(R_b))] \tag{13.3}$$

is the expected probability of success (as assessed by investors).

The key point is that this average probability of success is increasing in R_b: the lower the debt burden (the higher R_b is), the more accountable the entrepreneur is. An increase in the interest rate r increases the debt burden for a given $\hat{p}$ (see equation (13.2)): R_b decreases, making the entrepreneur less accountable (see equation (13.1): B^* decreases), which in turn increases the debt burden, and so forth. This vicious circle may lead to a discontinuous drop (collapse, breakdown) in lending.

Example. Suppose that $p_L = 0$, $\bar{B} = 1$, and H is uniform on $[0, 1]$: $H(B) = B$. Then $H(B^*) = B^* = (\Delta p)R_b = p_H R_b$, and so

$$\hat{p}(R_b) = p_H B^* + p_L(1 - B^*) = p_H^2 R_b.$$

The investors' breakeven condition is

$$p_H^2 R_b(R - R_b) = (1+r)(I-A).$$

The possibility of collapse is illustrated in Figure 13.3. When the interest rate is equal to r_0, there are two possible equilibria, $\hat{R}_b$ and R_b^*. Both satisfy the investors' breakeven condition. The socially optimal one is the one that is preferred by entrepreneurs and yields entrepreneurial stake R_b^*. (It is also the only stable equilibrium: starting from entrepreneurial stake $\hat{R}_b$, a small increase in R_b raises $\hat{p}$ proportionally more than $(R - R_b)$ decreases, and so investors' profit increases, increasing R_b further, and so forth.) A small increase in r_1 completely shuts down the credit market.[12]

(b) *Impact on the pool of applicants: asymmetric information about profitability.* Still in the fixed-investment model, assume now that loan applicants differ in their probability of success rather than in their private benefit. So B is the same for all entrepreneurs, but the probability of success is

$$p + \tau;$$

that is, the probability of success is $p_H + \tau$ if the entrepreneur behaves and $p_L + \tau$ if she misbehaves. As usual, the benefit of this separable form is that incentives can be separated from adverse selection, since the incentive constraint,

$$[(p_H + \tau) - (p_L + \tau)]R_b \geqslant B,$$

for a contract {R_b in the case of success, 0 in the case of failure} is independent of τ.

The profitability parameter τ is distributed according to some cumulative distribution function $H(\tau)$ with density $h(\tau)$ on $[\underline{\tau}, \bar{\tau}]$ (we keep the same notation $H(\cdot)$ for the distribution of the privately known parameter, here τ). Let[13]

$$\tau^+(\tau) \equiv E(\tilde{\tau} \mid \tilde{\tau} \geqslant \tau) = \frac{\int_\tau^{\bar{\tau}} \tilde{\tau} h(\tilde{\tau})\,\mathrm{d}\tilde{\tau}}{1 - H(\tau)}$$

and

$$\tau^-(\tau) \equiv E(\tilde{\tau} \mid \tilde{\tau} < \tau) = \frac{\int_{\underline{\tau}}^{\tau} \tilde{\tau} h(\tilde{\tau})\,\mathrm{d}\tilde{\tau}}{H(\tau)}$$

denote the truncated means. For example, $\tau^+(\tau)$ is the expectation of $\tilde{\tau}$ conditional on $\tilde{\tau}$ exceeding τ.

To simplify the exposition, we will assume that the entrepreneur has no cash on hand:

$$A = 0.$$

But we will allow for a general reservation utility $\bar{U}_b(\tau)$ for the entrepreneur. Until now, we have mostly assumed that the reservation utility is type-independent:

$$\bar{U}_b(\tau) = \bar{U}_b \quad \text{for all } \tau$$

(and have normalized the reservation utility to be 0: $\bar{U}_b = 0$).

We will also be interested in situations in which the utility corresponding to the "outside option," $\bar{U}_b$, increases with τ, and possibly steeply so (the case in which $\bar{U}_b$ increases little with τ is qualitatively similar to that in which it is constant). For example, a talented researcher may have excellent academic prospects (the outside option) when contemplating switching careers and raising funds for a start-up. Or, if the financing helps the firm strengthen its productive capacity or expand, a firm with a good project has a better "outside option" (not being refinanced).[14]

Remark (absence of reward for failure). In the discussion of the incentive constraint above, we assumed that the entrepreneur receives 0 in the case of failure. This is indeed what moral-hazard considerations dictate. But adverse selection only reinforces the optimality of the absence of reward in the case of failure, since such rewards tend to "screen in" low-profitability entrepreneurs. Hence, competitive investors are wary of such contracts.[15] The absence

12. This insight is less interesting than the previous observation that moral hazard increases with the rate of interest; for, the possibility that the market shuts down completely as the interest rate increases slightly also arises under symmetric information: when B is known, and in the absence of other sources of heterogeneity, the market for loans shuts down when r reaches r^*, where

$$(1 + r^*)(I - A) = p_H\left(R - \frac{B}{\Delta p}\right).$$

The basic point, though, is that the introduction of heterogeneity (here with respect to the private benefit) does not eliminate discontinuous market breakdowns.

13. It is well-known that τ^+ and τ^- both grow with τ, at a rate between 0 and 1 as long as the distribution's hazard rates $h/(1-H)$ and h/H are, respectively, increasing and decreasing (see, for example, An 1998).

14. For a state-of-the-art study of contracting under type-dependent outside options, see Jullien (2000).

15. Suppose that type τ selects a scheme $\{R_b^S(\tau), R_b^F(\tau)\}$ describing the rewards in the cases of success and failure. Assuming that contracts inducing misbehavior yield a negative NPV, and therefore focusing without loss of generality on contracts that do not induce shirking ($R_b^S(\tau) - R_b^F(\tau) \geqslant B/\Delta p$ for all τ), type τ will choose the contract that is most appropriate for the type, and so solves

$$\max_{\hat{\tau} \in [\underline{\tau}, \bar{\tau}]} \{(p_H + \tau)R_b^S(\hat{\tau}) + (1 - p_H - \tau)R_b^F(\hat{\tau})\}.$$

A simple revealed-preference argument (write the two inequalities saying that type τ prefers $\{R_b^S(\tau), R_b^F(\tau)\}$ to $\{R_b^S(\tau'), R_b^F(\tau')\}$ and conversely for type τ' and add up the two inequalities) yields

$$(\tau' - \tau)[[R_b^S(\tau') - R_b^F(\tau')] - [R_b^S(\tau) - R_b^F(\tau)]] \geqslant 0$$

for all (τ, τ').

Note, finally, that incentive compatibility in the choice of contracts implies that a borrower cannot get more for both realizations than of reward in the case of failure implies that a contract is solely described by the reward R_b in the case of success.

Case 1. High-profitability entrepreneurs are more eager to receive funding. Assume in a first step that the reservation utility does not depend on type (or more generally does not grow fast with the entrepreneur's type):

$$\bar{U}_b(\tau) = \bar{U}_b.$$

Then, for a given R_b, only entrepreneurs with type $\tau \geqslant \tau^*(R_b)$ apply for funding, where

$$[p_H + \tau^*(R_b)]R_b = \bar{U}_b, \tag{13.4}$$

because the utility from the project, $(p_H + \tau)R_b$, is increasing in profitability. The investors' expected income is then

$$[p_H + \tau^+(\tau^*(R_b))](R - R_b).$$

And so the investors' breakeven condition for a given market rate of interest r is[16]

$$[p_H + \tau^+(\tau^*(R_b))](R - R_b) = (1 + r)(I - A). \tag{13.5}$$

Note that the left-hand side of (13.5) decreases with R_b. Thus, keeping the pool of applicants constant, an increase in the interest rate leads to an increased stake demanded by investors (R_b decreases[17]), which in turn improves the pool of applicants (τ^* increases).

Case 2. *Low-profitability entrepreneurs are more eager to receive funding.* Suppose now that $\bar{U}_b(\tau)$ is "steeply increasing" (meaning: it is increasing faster than the utility obtained from receiving funding[18]). The contract R_b then attracts the *worst* types:

$$\tau \leqslant \tau^*(R_b),$$

where

$$[p_H + \tau^*(R_b)]R_b = \bar{U}_b(\tau^*(R_b)). \tag{13.6}$$

The investors' breakeven condition is then given by

$$[p_H + \tau^-(\tau^*(R_b))](R - R_b) = (1 + r)(I - A). \tag{13.7}$$

An increase in the rate of interest now has a drastically different impact. As in case 1, the direct effect is to increase the debt burden $(R - R_b)$. But condition (13.6), together with the fact that $\bar{U}_b(\cdot)$ is steeply increasing, implies that τ^* decreases (the pool of applicants worsens), which lowers τ^-, leading to a further increase in $(R - R_b)$. This spiral may lead to a complete collapse of the credit market.[19]

The two cases are illustrated in Figure 13.4.

13.3 Loanable Funds and the Credit Crunch: The Lending Channel

13.3.1 A "Double-Decker" Model

As was discussed in the introduction to this chapter, firms in the productive sector may not be hit solely by their own capital shortage (the *balance-sheet channel*), but also by a weakness in the balance sheets of the financial institutions that lend to them (the *lending channel*).

A credit crunch refers to a situation in which the banks' equity has fallen to a low level and so banks are capital constrained and cannot lend as much

another borrower:

$$R_b^S(\tau') < R_b^S(\tau) \quad \text{if } R_b^F(\tau') > R_b^F(\tau).$$

And so contracts that offer a higher reward for failure (and so by incentive compatibility embody a smaller wedge $R_b^S(\cdot) - R_b^F(\cdot)$) attract lower-profitability types.

16. We are a bit informal here. To be more rigorous, we need to specify whether the entrepreneur selects R_b or investors compete to obtain the entrepreneur's business (the answer is the same for both cases). For example, if the investors compete, for the candidate equilibrium described by (13.4) and (13.5), an investor offering a lower R_b would not interest the entrepreneur, while one offering a higher $R_b' > R_b$ would attract a worse pool of applicants, namely, those with type $\tau \geqslant \tau'$, where $\tau' < \tau^*$ is given by $(p_H + \tau')R_b' = \bar{U}_b$. Hence, this investor would have both a smaller stake *and* a lower probability of success.

17. At least as long as the entrepreneur's reward is sufficient to deter shirking.

18. Again, we are a bit informal here, since the latter utility grows with τ at rate R_b, where R_b is endogenous. It is straightforward to be more careful (note in particular that $R_b \leqslant R$), but we leave this to the reader for the sake of conciseness.

19. Let us illustrate the possibility of a collapse. Suppose that τ is distributed uniformly on $[0, \bar{\tau}]$. And so $\tau^-(\tau^*) = \frac{1}{2}\tau^*$. Let $\bar{U}_b(\tau) = K\tau$, where $K \geqslant R$ (and so the reservation utility grows faster with τ than the utility from being funded, which itself grows at rate $R_b < R$).

Then, for a given $R_b \in [B/\Delta p, R]$, the threshold $\tau^*(R_b)$ under which the entrepreneur applies for funding is given by

$$[p_H + \tau^*(R_b)]R_b = K\tau^*(R_b).$$

The investors' breakeven condition is therefore

$$\left[p_H + \frac{p_H R_b}{2(K - R_b)}\right](R - R_b) = (1 + r)(I - A).$$

It is straightforward to construct examples in which a small increase in the interest rate shuts down a hitherto sizeable loan market.

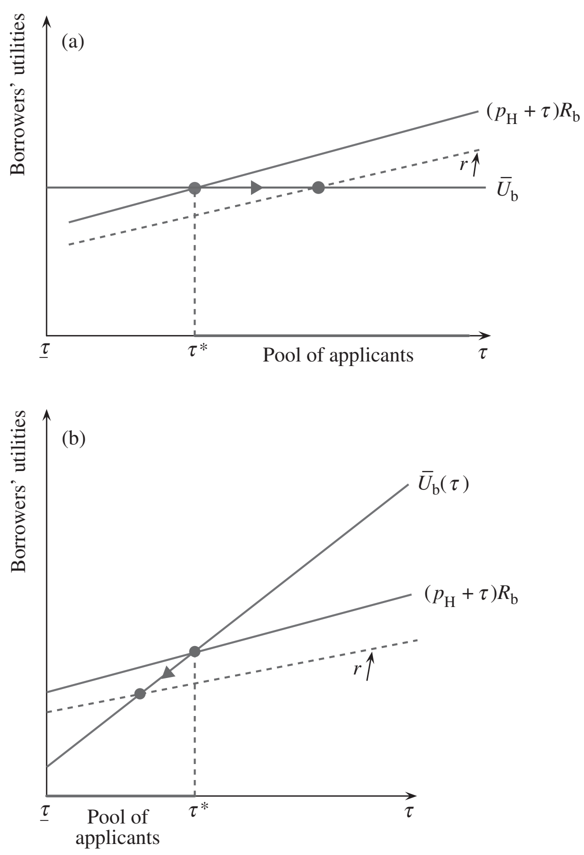


Figure 13.4 (a) Case 1; (b) case 2.

as the opportunities offered to them would warrant.[20] But, of course, the phenomenon has broader applicability: whenever borrowers need to resort to what was labeled "informed capital" in Chapter 9, namely, investors who play a monitoring or certification function, a weakness in the balance sheet of the latter translates into difficult times for the former. For example, a capital shortage at the venture capitalists' level translates one tier down into an increased difficulty for start-ups to raise funds.

This discussion suggests taking a "double-decker" view of credit rationing: the same logic that limits the availability of credit for the "real sector" firms also limits, one tier above, the ability of financial institutions to lend to these firms. Our treatment, which basically combines the partial equilibrium analysis of monitoring of Chapter 9 with the equilibrium approach of Section 13.1, follows Holmström and Tirole (1997).[21]

We thus consider three risk-neutral groups of economic agents: borrowers (firms), monitors (banks), and ordinary (uninformed) investors.

We will assume that each group is composed of a continuum of members, and so market power issues do not arise. The description of equilibrium will distinguish between two rates of interest or rates of return:

(i) the rate demanded by investors—we will let γ denote one plus this rate of interest (so $\gamma = 1 + r$ in the notation of Section 13.1); and
(ii) the rate demanded by monitors on their own invested funds—we will let χ denote one plus this rate of interest.

In equilibrium,

$$\chi > \gamma$$

for two reasons: the first is that monitors must be compensated for their monitoring cost, a cost not incurred by ordinary investors. Because monitors can always decide to invest as ordinary investors, it must be the case that they indeed get a superior return if they are induced to monitor (more on this below). Second, and more interestingly, χ may embody a scarcity rent. If the demand for monitoring is large compared with the supply, then banks are able to extract quasi-rents by charging a high rate of interest.[22]

As in Section 13.1, we consider both the fixed- and the variable-investment variants.

13.3.2 Fixed Investment Size

Entrepreneurs. There is a continuum of entrepreneurs/firms of mass 1. Each has one potential project of size I, yielding profit R in the case of success and 0 in the case of failure. As in Chapter 9, we

20. In the case of banks, capital adequacy requirements set by the Basel Committee and enforced by National Regulatory Authorities directly or indirectly (through the fear of a later constraint) constrain the amount that poorly capitalized banks can lend. Similar regulations apply to insurance companies (see, for example, Dewatripont and Tirole 1994).

21. See also Repullo and Suarez (2000), who look at the impact of monetary shocks (modeled as shifts in the riskless interest rate).

22. This is unrelated to the exercise of market power, since we have assumed there was none.

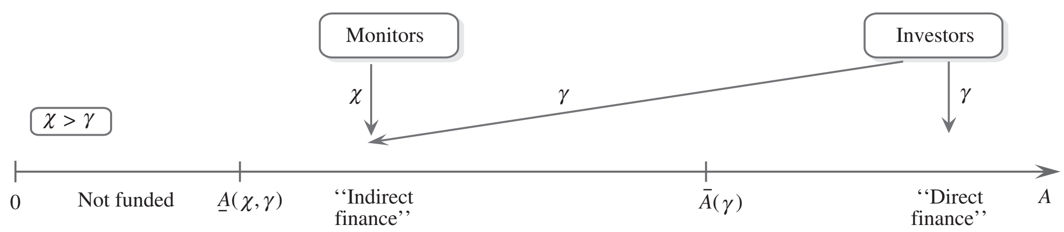


Figure 13.5 Certification.

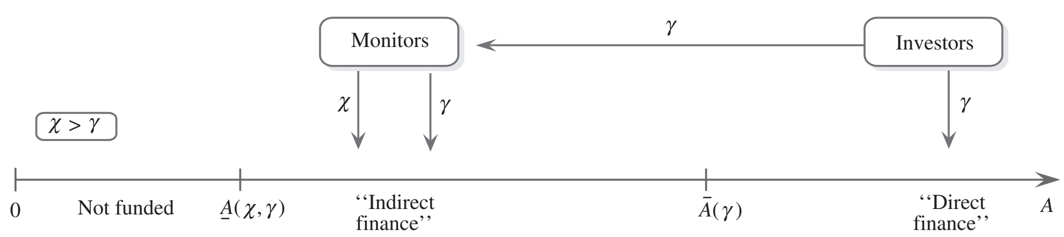


Figure 13.6 Intermediation.

assume that there are three versions of the project:

	good	bad	Bad
Pr(success)	p_H	p_L	p_L
Private benefit	0	b	B

with $p_H = p_L + \Delta p > p_L$ and $B > b > 0$.

Only the good version (good behavior) delivers a positive NPV when financed by uninformed investors:

$$p_H R - \gamma I > 0 > [p_L R - \gamma I] + B.$$

Entrepreneurial heterogeneity can be modeled in a number of ways. Let us assume here that entrepreneurs differ in their net worth, A. Net worth is distributed according to cumulative distribution $G(A)$ on $[0, \infty)$.

There are two categories of (risk-neutral) investors:

Monitors (financial intermediaries, banks). As in Chapter 9, a monitor can at monitoring cost c rule out the Bad project (the one with high private benefit B). As for the entrepreneur's private benefit, the monitor's cost c, if any, is incurred in the second period. There is a continuum of monitors, with total net worth K_m (the distribution, under some assumptions, turns out to be irrelevant). They demand rate of return χ on their (own-account) investment.

Uninformed investors are individually small; they therefore free-ride in the monitoring activity and remain uninformed. As stated above, they demand expected rate of return γ.

We will say that the entrepreneur resorts to "direct or uninformed" finance if she borrows solely from uninformed investors, and to "indirect or informed" finance if a monitor is enlisted as well.

We can consider two cases:

- Exogenous interest rate: uninformed investors have access to a "storage facility" yielding γ units of good for each unit of investment. Their savings are completely elastic at interest rate $(\gamma - 1)$.
- Endogenous interest rate: the uninformed investors' savings are equal to $S(\gamma)$, where S is increasing in γ.

Let us begin with the case of an exogenous interest rate. The market equilibrium can be described in either of two equivalent ways: certification (Figure 13.5) and intermediation (Figure 13.6).

Intermediation occurs when the monitor collects funds from uninformed investors and offers to entrepreneurs bundled loans using both their own capital and the money collected from uninformed investors. For example, banks collect deposits from depositors and lend these as well as bank capital to firms. By contrast, a venture capitalist or a lead investment bank put their own funds into the borrowers' ventures, which then attract (at a different rate of return) the funds of less informed investors (junior partners, say). It is clear that the choice of denomination, in our simple-minded model, is a pure matter of accounting of investment flows and has no real economic implication.

Without a monitor, the borrower, when financed, obtains NPV

$$U_b^* \equiv \frac{p_H R}{\gamma} - I.$$

The entrepreneur's ability to raise uninformed finance as usual depends on her ability to generate enough pledgeable income to reimburse the uninformed investors' initial outlay. Let the revenue R in the case of success be shared between the borrower (R_b) and the uninformed investors (R_u).

The financing condition,

$$p_H R_u \geqslant \gamma(I - A),$$

and the incentive compatibility condition,

$$(\Delta p) R_b \geqslant B$$

(where we use the fact that the borrower prefers the Bad project to the bad project when misbehaving), must both be satisfied. And so

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant \gamma(I - A)$$

or

$$A \geqslant \bar{A}(\gamma) \equiv I - \frac{p_H}{\gamma}\left[R - \frac{B}{\Delta p}\right].$$

When $A < \bar{A}(\gamma)$, the entrepreneur cannot obtain financing, at least in the absence of a monitor. The cutoff $\bar{A}(\gamma)$ increases with γ.

With a monitor, we will use the certification paradigm, which is conceptually slightly simpler. The revenue in the case of success is then divided among borrower (R_b), uninformed investors (R_u), and monitor (R_m). On the investment side, the borrower brings A, the monitor I_m, and the uninformed investors $I_u = I - A - I_m$.

Note that by definition of the rate of return χ demanded by the monitor, the following accounting identity prevails:

$$p_H R_m = \chi I_m.$$

Similarly,

$$p_H R_u = \gamma I_u.$$

The entrepreneur's net utility, given that she could invest A at the market rate of return γ, is then

$$\begin{aligned} U_b &= \frac{p_H(R - R_m - R_u)}{\gamma} - A \\ &= \frac{p_H R - \chi I_m - \gamma I_u}{\gamma} - A \\ &= \frac{p_H R - (\chi - \gamma) I_m}{\gamma} - I. \end{aligned}$$

Recall our intuition that χ exceeds γ. One reason for this, as we have noted, is that the monitor could choose to be an uninformed investor in other firms and economize the monitoring cost c; so

$$\chi I_m - c \geqslant \gamma I_m \quad \text{or} \quad \chi - \gamma \geqslant c/I_m.$$

We thus conclude that $U_b^* > U_b$ and so the entrepreneur is better off dispensing with a monitor if she can afford to, i.e., if $A \geqslant \bar{A}(\gamma)$; and that for $A < \bar{A}(\gamma)$, she will want to minimize the monitor's capital involvement I_m.

Suppose that

$$(\Delta p) R_b < B$$

(otherwise the entrepreneur would not need to be monitored), but

$$(\Delta p) R_b \geqslant b,$$

so that, when monitored, the entrepreneur is induced to behave. The monitor's incentive compatibility constraint is then

$$(\Delta p) R_m \geqslant c.$$

This minimum stake in turn requires a minimum investment:

$$I_m \geqslant I_m(\chi) \equiv \frac{p_H c}{(\Delta p)\chi}.$$

Note that the minimum acceptable rate of return for monitors (given by $(\chi - \gamma) I_m = c$) satisfies $\chi = p_H \gamma / p_L$.

The entrepreneur can leverage the presence of a monitor to obtain financing if and only if the present discounted income that can be pledged to the uninformed investors exceeds their initial outlay, or

$$\frac{p_H(R - (b + c)/\Delta p)}{\gamma} \geqslant I - A - I_m(\chi),$$

or

$$A \geqslant \underline{A}(\gamma, \chi),$$

where $\underline{A}(\gamma,\chi)$ is increasing in γ and χ. Because $\chi > \gamma$,

$$\underline{A}(\gamma,\chi) < \bar{A}(\gamma)$$

if and only if $c < \bar{c}$ (with some $\bar{c} > 0$),[23] which we will assume. It must also be the case that entrepreneurs prefer to enlist a monitor and receive funding rather than just invest their net wealth in other firms, i.e., that their net utility is positive:

$$\frac{p_{\mathrm{H}}R - (\chi-\gamma)p_{\mathrm{H}}c/(\Delta p)\chi}{\gamma} \geqslant I. \tag{13.8}$$

If the monitor obtains no rent from monitoring ($(\chi-\gamma)I_{\mathrm{m}} = c$), then condition (13.8) boils down to

$$p_{\mathrm{H}}R - c \geqslant \gamma I. \tag{13.9}$$

When the monitor receives a rent ($(\chi-\gamma)I_{\mathrm{m}} > c$), condition (13.8) is more stringent than (13.9). Note, however, that if (13.8) were violated, then there would be no demand for monitoring capital and so monitors could not obtain rents after all. Inequality (13.9) is then the relevant condition.

To complete the description of equilibrium, we equate supply of and demand for informed capital:

$$K_{\mathrm{m}} \geqslant [G(\bar{A}(\gamma)) - G(\underline{A}(\gamma,\chi))]I_{\mathrm{m}}(\chi), \tag{13.10}$$

with inequality only if $(\chi-\gamma)I_{\mathrm{m}}(\chi) = c$.

When the interest rate is endogenous, the rates of return γ and χ must also clear the savings market:[24]

$$\begin{aligned} S(\gamma) = \int_{\bar{A}(\gamma)}^{\infty} (I-A)\,\mathrm{d}G(A) \\ + \int_{\underline{A}(\gamma,\chi)}^{\bar{A}(\gamma)} [I - A - I_{\mathrm{m}}(\chi)]\,\mathrm{d}G(A) \\ - \int_{0}^{\underline{A}(\gamma,\chi)} A\,\mathrm{d}G(A). \end{aligned} \tag{13.11}$$

Note that entrepreneurs who have assets A in excess of investment I, if any,[25] do not need to borrow and actually reinvest the surplus $A - I$ in other firms.

The equilibrium rates of return (γ,χ) are then given by (13.10) and (13.11).

Turning to comparative statics (in the broadest framework in which the rate of return received by uninformed investors is endogenous), we can consider the impact of three types of recession:

(a) *Industrial recession (balance-sheet channel).* The distribution $G(A)$ shifts toward lower values of A (that is, $G(A)$ increases for all A). As in Section 13.2, the distribution G is indexed by a parameter θ of first-order stochastic dominance: $G(A \mid \theta)$ with $\partial G/\partial\theta < 0$. An industrial recession corresponds to a decrease in θ, i.e., to a less favorable distribution.

(b) *Credit crunch (lending channel).* K_{m} decreases.

(c) *Shortage of savings.* γ increases (in the perfectly elastic case) or S decreases.

It is easily shown (see Holmström and Tirole 1997) that *in the three types of capital squeeze, aggregate investment goes down and the threshold* $(\underline{A}(\gamma,\chi))$ *over which firms can raise financing increases.*

In particular, firms with weak balance sheets ($\underline{A} \leqslant A < \bar{A}$), which need access to intermediaries in order to raise financing, are hurt by a credit crunch: as monitoring capital K_{m} shrinks, the intermediaries demand a higher rate of return, χ, which squeezes out the marginal firms (with A just above $\underline{A}$) and hurts the others.[26] Firms with strong balance sheets, in contrast, are not directly affected since their financing does not depend on access to intermediaries. They may even gain in a credit crunch to the extent that the reduced demand for uninformed capital by weaker firms may lower the uninformed investors' rate of return. Concretely, banks may become greedier, while the rate of interest on bonds may fall.[27]

23. If $\bar{A} \leqslant \underline{A}$, then there is excess supply of monitoring capital and so $\chi = p_{\mathrm{H}}\gamma/p_{\mathrm{L}}$; thus $\bar{c}(\Delta p) = p_{\mathrm{H}}(B-b)$.

24. Again, the entrepreneurs who do not receive funding save. Holmström and Tirole (1997) implicitly assumed that those who do not get funding do not save, an assumption at odds with the assumption that those entrepreneurs who have cash on hand $A > I$ do save their excess cash (we are grateful to Flavio Toxvaerd for pointing this out to us). The results are qualitatively identical for the various assumptions that can be made about idle entrepreneur wealth.

25. Section 13.2.1 assumed for simplicity that the upper bound on A was lower than I. This assumption is really not crucial, as shown here.

26. Relatedly, Davies and Ioannidis (2003), looking at the behavior of bond issuance and bank lending in the United States between 1970 and 1999, find that securities issuance often does not offset a decline in bank lending, and thereby confirm that the different sources of finance are not substitutable.

27. Needless to say, stronger firms may not benefit from a credit crunch for reasons that are not modeled here. For example, productive activities may exhibit strategic complementarities, as has been emphasized in many macroeconomic models (e.g., Diamond 1982; Shleifer 1986; Cooper and John 1988; Matsuyama 1991).

Lastly, from (13.11), it is apparent that monitors enjoy a rent $((\chi - \gamma)I_{\mathrm{m}} > c)$ if and only if K_{m} lies below some threshold. Above that threshold, there is excess supply of monitoring capital and the monitors' rate of return is determined by their indifference between investing in firms they monitor and investing in a portfolio of other firms that they do not monitor.[28]

13.3.3 Variable Investment Size

For the sake of completeness, let us investigate the case of constant-returns-to-scale production. For investment I, a firm's income is RI in the case of success, and 0 in the case of failure; the private benefit is BI if left unmonitored and bI if monitored, in the case of entrepreneurial misbehavior (yielding probability of success p_{L}), and 0 in the case of good behavior (yielding probability of success p_{H}). The monitoring cost is also proportional to investment: cI. The cost of this constant-returns-to-scale modeling is that there are no longer firms with weak and strong balance sheets: firms are homogeneous up to a scaling factor (namely, their individual net worth A). As a corollary, only total entrepreneurial capital,

$$K_{\mathrm{b}} \equiv \int_0^\infty A \, \mathrm{d}G(A),$$

matters for the determination of equilibrium interest rates and activity, not its distribution among entrepreneurs.

Letting K denote total investment, and decomposing it among the contributions of borrowers, monitors, and uninformed investors,

$$K = K_{\mathrm{b}} + K_{\mathrm{m}} + K_{\mathrm{u}},$$

let

$$\mathfrak{r}_{\mathrm{m}} \equiv \frac{K_{\mathrm{m}}}{K_{\mathrm{m}} + K_{\mathrm{u}}} \quad \text{and} \quad \mathfrak{r}_{\mathrm{b}} \equiv \frac{K_{\mathrm{b}}}{K}.$$

The ratio of the monitors' own funds to total outside finance, $\mathfrak{r}_{\mathrm{m}}$, can be interpreted as the solvency ratio of the monitor under the intermediation paradigm.[29] And $\mathfrak{r}_{\mathrm{b}}$ is the equity ratio of the borrowers.

We leave it to the reader to check that

$$\begin{array}{c}\text{a \textit{credit crunch}}\\ \text{(reduction in } K_{\mathrm{m}})\end{array} \left\{\begin{array}{l}\text{decreases } \gamma,\\ \text{increases } \chi,\\ \text{decreases } \mathfrak{r}_{\mathrm{m}},\\ \text{increases } \mathfrak{r}_{\mathrm{b}};\end{array}\right.$$

$$\begin{array}{c}\text{a \textit{collateral squeeze}}\\ \text{(a decrease in } K_{\mathrm{b}})\end{array} \left\{\begin{array}{l}\text{decreases } \gamma,\\ \text{decreases } \chi,\\ \text{increases } \mathfrak{r}_{\mathrm{m}},\\ \text{decreases } \mathfrak{r}_{\mathrm{b}}.^{30}\end{array}\right.$$

Discussion. This simple model leaves a number of questions open. First, the equivalence of certification and intermediation, while a convenient feature, ought to be reexamined in broader setups. In practice, intermediation gives the intermediary more leeway in allocating the uninformed investors' funds. This leeway, unlike in this model, may aggravate moral hazard. On the other hand, the monitor can more easily enjoy the benefits of diversification under intermediation than under project finance, an issue which again does not arise in this basic model. Second, we have modeled intermediaries as being homogeneous, perhaps up to a scaling factor. In practice, there is a continuum of intermediaries with different monitoring intensities and accordingly with different stakes in the success of the firms they monitor.[31] Furthermore, the demand for various types of monitoring capital moves around with the economic cycle; in particular, firms that have gone through difficult times or face dim prospects need to resort to higher-intensity monitoring.

Third, and more importantly for the sake of this chapter, the story told here is inherently static. Comparative statics was performed on inherited levels of monitoring and entrepreneurial capitals. In practice,

28. We earlier stated that the distribution of K_{m} among intermediaries is irrelevant under some assumptions. Note, first, that individual intermediaries must invest $I_{\mathrm{m}}(\chi)$ in each of the monitored firms. One possibility is, thus, that each intermediary has capital equal to a multiple of $I_{\mathrm{m}}(\chi)$. If this "integer condition" is not satisfied, then some monitoring capital may be wasted. The analysis then becomes more cumbersome, but is not substantially altered. Second, and in reference to Section 4.2, if some individual intermediaries have more than $I_{\mathrm{m}}(\chi)$ and are each able to monitor multiple firms, then we implicitly rule out any ability to diversify. One may have in mind that intermediaries are specialized, in that the shocks faced by the firms they monitor are perfectly correlated (see Chapter 4). Again, this assumption is made for analytical convenience, and does not affect the analysis in a qualitative way.

29. The ratio $\mathfrak{r}_{\mathrm{m}}$ is a crude version of the Cooke ratio in banking regulation.

30. For completeness, a savings squeeze increases γ, decreases χ, increases $\mathfrak{r}_{\mathrm{m}}$ and $\mathfrak{r}_{\mathrm{b}}$.

31. An introduction to this issue can be found in Holmström and Tirole (1997).

there are subtle dynamic interactions between the two, with interesting leads and lags. A key item on the research agenda is to come up with a tractable dynamic version of this "double-decker model."

13.4 Dynamic Complementarities: Net Worth Effects, Poverty Traps, and the Financial Accelerator

This section returns to the "single-tier" structure (that is, it ignores monitoring and the issue of scarcity of monitoring capital studied in Section 13.3). It introduces dynamics and shows that corporate finance considerations lead to strong hysteresis effects[32] where there would be none in an (Arrow–Debreu) framework without agency cost.

13.4.1 Sources of Dynamic Complementarities

Two main sources of hysteresis have been studied in the literature.

Retained earnings/balance sheet effects. A firm coming out of a recession (with low profitability at date t) tends to lack resources to finance new investments. In the absence of agency cost, this lack of resources would have no impact on refinancing,[33] as forward-looking investors and managers would optimally focus on prospects and arrange the financing of positive-NPV projects. Not so in the presence of an agency cost. The latter creates scope for credit rationing, which implies that current profitability affects future investment and future activity (as we already observed in Chapter 5).

For example, if we assume that investments depreciate in one period and that the contracts between the firm and its investors are short-term contracts, in which the investors are repaid for their date-t investment out of the date-t profit (a strong assumption, as we noted in Chapter 5), and letting A_t, I_t, and y_t denote the assets, investment, and profit at date t, the mechanics of hysteresis can be schematically described in the following way:

$$y_t \to A_{t+1} \to I_{t+1} \to y_{t+1} \to A_{t+2} \to \cdots .$$

32. A hysteresis effect refers to the lagging of an effect behind its cause.

33. Unless the firm's low profitability at date t conveys negative information about its profitability at dates $t+1, t+2, \dots$.

New entrepreneurs' opportunities. Rather than focusing on balance sheet effects of *existing* firms, some models trace the source of hysteresis to the impact of existing activity on *would-be* entrepreneurs through factor prices. For example, these potential entrepreneurs may offer their labor to the incumbent firms before accumulating enough wealth to become entrepreneurs themselves. This idea is most easily analyzed in an overlapping-generations framework. In the two papers that initiated the literature on the topic—by Bernanke and Gertler (1989), the seminal formal study of the financial accelerator, and by Banerjee and Newman (1991, 1993)—the young work and thereby accumulate wealth, which they can use to start their own firm when they are older. A higher level of investment and activity at date t raises the demand for labor and thereby the wage w_t of laborers, who then have more resources, which facilitates their access to funding at date $t+1$. In Aghion and Bolton (1997), Piketty (1997), and Matsuyama (2000), by contrast, the effect operates through the interest rate rather than through wages.[34]

Relatedly, the literature has also emphasized the possibility that credit rationing traps either individuals (or families) or entire societies in poverty. We will accordingly provide examples of such individual and collective poverty traps.[35]

13.4.2 Dynamics of Wealth Distribution: A Tale of Two Families

First we provide an example of an individual (family) poverty trap. To develop this example, we will need the following preamble.

13.4.2.1 The Warm-Glow Model

We study long-lived lineages of short-lived family members. Parents become entrepreneurs if they

34. There are, of course, other reasons why current activity may affect the new entrepreneurs' ability to raise funding. For example, a high level of activity may increase tax receipts and boost public investment in infrastructure and thereby improve the profitability of new private investments. Or there may be spillovers and accumulation of social capital. But such sources of hysteresis are not related to corporate finance considerations (unless, say, the public infrastructure investment affects corporate governance, e.g., reduces B in the model).

35. See also Banerjee (2003) and Matsuyama (2005) for excellent discussions of poverty traps, including ones that are not based on credit rationing.

have sufficient funds, earn income, and finally leave wealth as a bequest to their children, who may then use this wealth to undertake projects of their own, and so forth. What motivates parents to leave money to their children is an important modeling choice. Parents who internalize the welfare of their children must also, at least indirectly, internalize that of their grandchildren, that of their great-grandchildren, and thus that of all members of the lineage. Their choice of bequest then resembles that of liquidity management by a long-lived individual unable to secure long-term finance (i.e., facing a sequence of short-term borrowing deals) (see Section 4.7.2). Such liquidity management is complex. For the purposes of this section, we first bypass the difficulty in a somewhat ad hoc way by using the warm-glow model, which enables us to discuss dynamics without worrying about dynamic programming. Namely, suppose that the following conditions hold.

- Individuals live for one period. An individual living at date t has exactly one heir who lives at date $t+1$, and so on.
- Individuals are "altruistic" in a rather specific way. Rather than caring about the utility of their heirs, they derive utility from the bequest they make to their heirs. We will assume that a generation-t individual derives utility from her own consumption c_t and from the bequest $\mathcal{L}_t$ to her heir.[36] Assume further that the utility function is a Cobb–Douglas utility function:

$$\left(\frac{c_t}{1-a}\right)^{1-a}\left(\frac{\mathcal{L}_t}{a}\right)^{a},$$

where $a \in (0,1)$ is the (impure) altruism parameter.[37]

Then the utility from income y_t is (taking logs)

$$\log U_t(y_t) = \max_{\{c_t,\mathcal{L}_t\}} \{(1-a)\log c_t + a \log \mathcal{L}_t\}$$

s.t.

$$c_t + \mathcal{L}_t = y_t.$$

This yields

$$c_t = (1-a)y_t \quad \text{and} \quad \mathcal{L}_t = a y_t,$$

and so

$$U_t(y_t) = y_t.$$

This formulation is particularly convenient since it allows us to keep our risk-neutral framework.

13.4.2.2 Lineages of Entrepreneurs in the Warm-Glow Model

Let us now consider a "warm-glow lineage" in which each generation t is a would-be entrepreneur, who

- is born with some exogenous endowment $\hat{A}$, to which is added the bequest $\mathcal{L}_{t-1}$ made by generation $t-1$;
- invests this total asset either in a storage technology yielding an interest rate equal to 0 (i.e., preserving the wealth) or in a fixed-size project as described in Section 13.2.1; and
- finally uses the proceed of this investment (her "income") for consumption c_t and bequest $\mathcal{L}_t$ to the next generation.

The timing is summarized in Figure 13.7.

Let us assume that the intraperiod rate of interest in the economy is equal to 0.[38]

As in the rest of the book, the private benefit B obtained by misbehaving is expressed in terms of money. So in the case of misbehavior the entrepreneur's utility from income y_t and private benefit B is $y_t + B$. As usual, we will assume that investment has a positive NPV if and only if the entrepreneur is induced to behave:

$$p_H R > I > p_L R + B.$$

It will also prove convenient to assume that success is a sure thing in the case of good behavior:[39]

$$p_H = 1.$$

36. We do not use the notation "B_t" for bequest in order not to create confusion with private benefits. Rather, we build on the French terminology for bequest ("legs").

37. This modeling borrows from Aghion and Bolton (1997), Andreoni (1989), Banerjee (2002), Banerjee and Newman (1991, 1993), Galor and Zeira (1993), Matsuyama (2000, 2002), and Piketty (1997). See Bénabou and Tirole (2005) and the references therein for a discussion of the various motives behind altruistic and prosocial behaviors.

38. For example, there might be outside investors demanding a rate of interest equal to 0; or else there are enough would-be entrepreneurs who do not make it to entrepreneurship and are indifferent between using the storage technology and lending to entrepreneurs at rate of interest equal to 0.

39. If we did not make this assumption, then, under the assumptions made below, the fraction of entrepreneurs in the population would converge to 0 as t goes to ∞ since failing entrepreneurs would deprive their heirs of the opportunity to become entrepreneurs (to avoid this, one could for example assume that $\hat{A}$ is stochastic).

Of course, when $p_H = 1$, the limited liability assumption cannot be motivated by large risk aversion for negative incomes. Relatedly, stiff

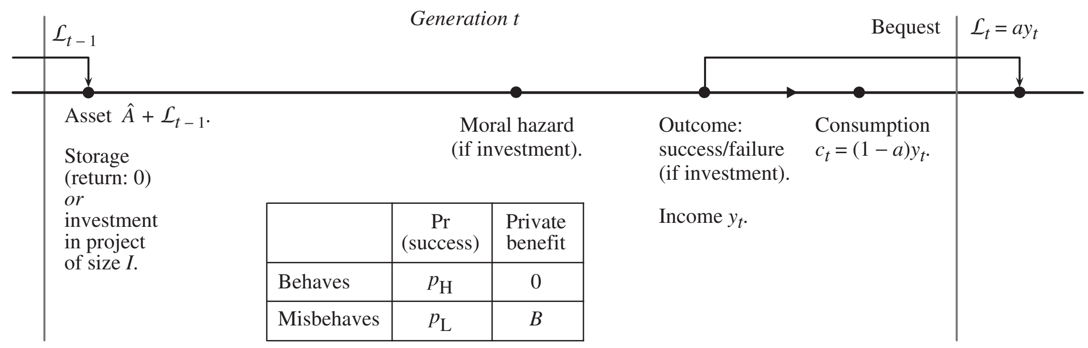


Figure 13.7

Lastly, let $\bar{A}$ be defined (as in equation (13.4)) by the equality between the pledgeable income and the investors' outlay:

$$p_H\left(R - \frac{B}{\Delta p}\right) = I - \bar{A}$$

or (using $p_H = 1$)

$$\bar{A} = I - \left(R - \frac{B}{\Delta p}\right).$$

The role of the following assumption will become clear shortly.

Assumption 13.1.

$$\frac{\hat{A} + a(R-I)}{1-a} > \bar{A} > \frac{\hat{A}}{1-a}.$$

(a) *Lineage stuck in a poverty trap.* Suppose that generation t receives bequest

$$\mathcal{L}_{t-1} < \bar{A} - \hat{A}.$$

Generation t's total wealth is then insufficient to have access to funds. Generation t must therefore invest $[\hat{A} + \mathcal{L}_{t-1}]$ into the low-return storage technology, and so

$$y_t = \hat{A} + \mathcal{L}_{t-1}.$$

With warm-glow preferences, bequests to generation $t+1$ are

$$\mathcal{L}_t = a y_t = a(\hat{A} + \mathcal{L}_{t-1}),$$

and so generation $t+1$ starts with

$$\hat{A} + \mathcal{L}_t = (1+a)\hat{A} + a\mathcal{L}_{t-1} < (1+a)\hat{A} + a(\bar{A} - \hat{A})$$

or

$$\hat{A} + \mathcal{L}_t < \hat{A} + a\bar{A} < \bar{A}$$

from Assumption 13.1.

The dynasty's total wealth per generation converges to $A_\infty < \bar{A}$, given by

$$A_\infty = \hat{A} + aA_\infty \quad \text{or} \quad A_\infty = \frac{\hat{A}}{1-a} < \bar{A}.$$

The lineage is stuck in a poverty trap.

(b) *Rich, entrepreneurial lineage.* By contrast, suppose that generation t's initial wealth exceeds $\bar{A}$:

$$\mathcal{L}_{t-1} > \bar{A} - \hat{A} \quad \text{or} \quad A_t \equiv \hat{A} + \mathcal{L}_{t-1} > \bar{A}.$$

Generation t has enough pledgeable income to offset the investors' outlay $I - (\hat{A} + \mathcal{L}_{t-1})$. Under risk neutrality, generation t selects the highest NPV solution and therefore prefers becoming an entrepreneur to investing in the storage technology. The NPV is then

$$p_H R - I = R - I > 0,$$

and so the entrepreneur's end-of-period income after reimbursing lenders is

$$y_t = (R - I) + A_t$$

(recall that the capital market is competitive, and so the entire NPV goes to the entrepreneur).

Generation $(t+1)$'s total wealth at the beginning of period $t+1$ is therefore

$$A_{t+1} = \hat{A} + a(R - I + A_t).$$

Note that

$$A_{t+1} > \hat{A} + a(R - I + \bar{A}) > \bar{A}$$

from Assumption 13.1.

jail sentences for defaulting entrepreneurs would be optimal and solve the moral-hazard problem. Thus, the case $p_H = 1$ is best viewed as an approximation of economies in which p_H is large, but smaller than 1.

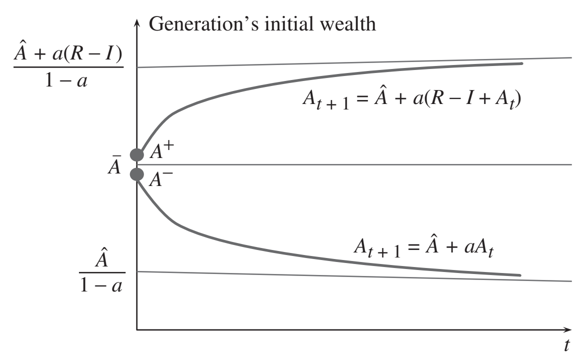


Figure 13.8

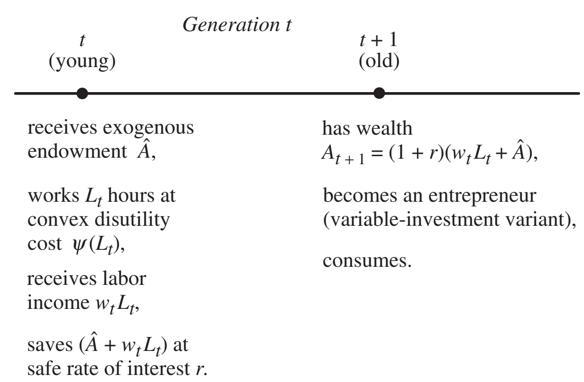


Figure 13.9

Hence, future generations also have the opportunity to become entrepreneurs.

The lineage's beginning-of-period wealth converges to A_∞, where

$$A_\infty = \hat{A} + a(R - I + A_\infty)$$

or

$$A_\infty = \frac{\hat{A} + a(R - I)}{1 - a}.$$

Figure 13.8 illustrates cases (a) and (b) and shows that a small difference in initial wealth (points A^- and A^+, respectively, in Figure 13.8) can make a big difference: for the current generation (as we know from Chapter 3) and even more for further generations.

Note, finally, that in the Arrow–Debreu world of no agency cost ($B = 0$), long-term incomes of different lineages would converge to $[\hat{A} + a(R - I)]/(1 - a)$ regardless of the lineage's initial wealth (rather than diverge as in Figure 13.8). A stronger investor protection, for example, reduces the dependency on wealth and generates a more equal long-run income distribution.

Discussion of the warm-glow assumption. The warm-glow model does not depict true altruism since each generation does not perfectly internalize the welfare of the next generation. Rather, individuals are portrayed as deriving utility from feeling or looking generous; they care about what they give rather than about how useful this gift is to the next generation.

This impure-altruism assumption turns out to be rather important for the treatment above. By contrast, consider pure altruism: generation $(t - 1)$ cares directly about generation t's welfare, U_t, rather than about the bequest $\mathcal{L}_{t-1}$. Then, starting at point A^- in Figure 13.8, a small increase in the bequest moving generation t's initial wealth to point A^+ increases U_t discontinuously, and so we would expect generation $(t-1)$ to increase its bequest so as to enable generation t to have access to financing. (This reasoning assumes that generations $t, t+1, \dots$ still have warm-glow utilities. If they themselves are truly altruistic, the analysis has to be modified slightly, because the incentive compatibility constraints are a bit different—on this, see also the treatment in Section 4.7—but the basic insight is unaltered.)

13.4.3 Collective Poverty Traps

As was pointed out earlier, hysteresis due to financial imperfections may occur at the level of a society, and not only at that of a family. We here pursue the wage conduit (Banerjee and Newman 1993).

Consider an overlapping-generations model in which

- a generation lives for two periods,
- young agents work and accumulate wealth,
- old agents are entrepreneurs and consume.

The timing for generation t is described in Figure 13.9, which is rather self-explanatory. A few further details, though:

- The rate of interest, r, from one period to the next is exogenous.
- The technology available to (old) entrepreneurs is the variable-investment model of Section 3.4,

reviewed in Section 13.2 (with as usual $p_{\mathrm{H}}R > 1 > p_{\mathrm{H}}(R - B/\Delta p)$). Investment, effort, outcome, and consumption all occur within period $t+1$.

- Producing output requires 1 unit of labor per unit of investment (the technology is a Leontief one, in which factors are combined in fixed proportions).
- The population is constant. Hence, the number of young and old agents are equal at any given point in time.
- Generation t's utility is $-\psi(L_t) + c_{t+1}$, where c_{t+1} is its consumption when old.
- The disutility of labor satisfies $\psi(0) = \psi'(0) = 0$, $\psi' > 0$, $\psi'' > 0$.

The assumption that more investment requires more labor (one-for-one in this example) drives hysteresis: a higher wealth accumulation in the past together with capital market imperfections raises investment, and therefore increases the demand for labor and the wage as well. A higher wage results in higher wealth accumulation, more investment, and so forth.

Let us focus on *steady states.*

Consider first an entrepreneur. A generation-t agent becomes an entrepreneur at date $t+1$. She then invests I_{t+1} and receives the NPV:

$$U_{\mathrm{b}}^{t+1} = [p_{\mathrm{H}}R - (1+w)]I_{t+1},$$

since now the unit cost includes the wage, w, per unit of investment.

The investment I_{t+1} is determined by the investors' breakeven condition:

$$(1+w)I_{t+1} - A_{t+1} \equiv p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right)I_{t+1},$$

where

$$A_{t+1} = (1+r)(wL_t + \hat{A})$$

is the wealth when old. Hence, the entrepreneur expected date-$(t+1)$ consumption is

$$U_{\mathrm{b}}^{t+1} = \left[\frac{p_{\mathrm{H}}R - (1+w)}{(1+w) - p_{\mathrm{H}}(R - B/\Delta p)}\right]A_{t+1}.$$

As expected, a higher labor cost w reduces both the NPV per unit of investment (the numerator in the fraction) and the borrowing capacity (through the denominator).

Let us now solve for the labor supply. The marginal cost at t, $\psi'(L_t)$, must equal the marginal benefit at $t+1$:

$$\frac{\mathrm{d}U_{\mathrm{b}}^{t+1}}{\mathrm{d}L_t} = \left[\frac{\mathrm{d}U_{\mathrm{b}}^{t+1}}{\mathrm{d}A_{t+1}}\right]\left[\frac{\mathrm{d}A_{t+1}}{\mathrm{d}L_t}\right].$$

Because

$$\frac{\mathrm{d}A_{t+1}}{\mathrm{d}L_t} = (1+r)w,$$

$$\psi'(L_t) = \left[\frac{p_{\mathrm{H}}R - (1+w)}{(1+w) - p_{\mathrm{H}}(R - B/\Delta p)}\right](1+r)w.$$

In steady state, and because the technology is a Leontief one,

$$L_t = L = I = I_{t+1},$$

and so

$$\psi'\left(\frac{(1+r)\hat{A}}{1 - wr - p_{\mathrm{H}}(R - B/\Delta p)}\right) = \frac{p_{\mathrm{H}}R - (1+w)}{(1+w) - p_{\mathrm{H}}(R - B/\Delta p)}(1+r)w. \quad (13.12)$$

The left-hand side (LHS) of condition (13.12), is increasing in w (from a positive level at $w = 0$). Its right-hand side (RHS) is concave. The steady-state equilibria are depicted (for the case $\psi''' > 0$) in Figure 13.10.

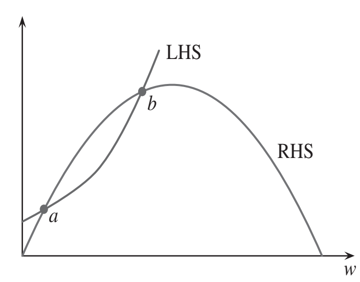


Figure 13.10

In Figure 13.10, there are two steady-state equilibria (there can be more generally). The wage and activity are higher in equilibrium b than in equilibrium a.[40] Cycles can also exist.

More on the literature. Matsuyama (2004) shows that heterogeneous technologies may be conducive

40. It is unclear in the absence of further assumptions whether, as long as they belong to the increasing part of the RHS, equilibria with higher wages dominate those with lower ones. For, a generation maximizes $\{-\psi(L) + (\mathrm{RHS})(L + \hat{A}/w)\}$ over L. Hence,

$$\frac{\mathrm{d}U_{\mathrm{b}}^{t+1}}{\mathrm{d}w} = \left(L + \frac{\hat{A}}{w}\right)\frac{\mathrm{d}(\mathrm{RHS})}{\mathrm{d}w} - \frac{\hat{A}(\mathrm{RHS})}{w^2}.$$

The second term represents the reduced multiplier on the agents' exogenous endowment.

to the existence of multiple steady-state equilibria and of cycles. In his model, as in Banerjee and Newman, economic agents accumulate wealth by supplying their labor in the first period of their life. Their wage income is then saved for the second period of their life, in which they can become lenders or entrepreneurs. Entrepreneurs produce units of physical capital, which, combined with labor, produce a final output. Matsuyama's model is built so that, despite credit market imperfections, there exists a unique equilibrium (similar to the neoclassical growth model equilibrium) when entrepreneurs face a unique production technology. Matsuyama then introduces a choice of technology in order to analyze composition effects. Suppose, for example, that there exist two technologies: a high-return/low-pledgeable-income technology and a low-return/high-pledgeable-income one. Multiple steady-state equilibria may then coexist: in a low-capital-intensity steady state, the wage of the young is low; and so their net worth when they build on that wage to become entrepreneurs is small. They consequently invest in a low-return/high-pledgeable-income technology that produces little capital. The dearth of capital generates a low wage for the next generation; and so forth. Matsuyama also demonstrates the possibility of credit cycles.

Aghion et al. (1999) explore the interest rate conduit and show how it may lead to real activity cycles. When entrepreneurs' borrowing capacity is low relative to savings, the interest rate falls. Entrepreneurs then need to reimburse less to allow investors to recoup interest and principal on their loans. Entrepreneurs then rebuild their net worth and increase their investments. This raises the demand for loans and puts pressure on the interest rate, increasing the entrepreneurs' debt burden, and so forth.[41]

13.5 Dynamic Substitutabilities: The Deflationary Impact of Past Investment

Section 13.4 emphasized dynamic complementarities: past investment raises the net worth of existing or would-be entrepreneurs, and thereby relaxes their current borrowing constraint, boosting investment today. Such dynamic complementarities can arise either at the level of families or at the country level.

By fixing the output price, though, the analysis of Section 13.4 neglected an obvious source of dynamic substitutability: in any given industry, an investment glut yesterday has a depressing effect on product prices and discourages investment today. This basic effect operates whether today's entrepreneurs are credit rationed or not; but under some circumstances, the contractionary impact is stronger when firms are credit rationed.

13.5.1 Heuristics

To obtain some first intuition as to how past investment crowds out current investment, let us start with a static model, with first a fixed investment, and then variable investment.

13.5.1.1 Fixed Investment Size

Consider, thus, the fixed-investment model. There is a mass 1 of entrepreneurs. At investment cost I, an entrepreneur can produce R units of a good with probability p (and 0 units with probability $1 - p$). The final price per unit of output is P. Presumably, P depends on past industry investment, but we do not need to go into detail at this stage. The probability of success is p_H if the entrepreneur behaves (no private benefit) and $p_L = p_H - \Delta p$ if she misbehaves (private benefit B). We assume that the output realizations are independent across entrepreneurs (there is no aggregate uncertainty); this assumption is consistent with the assumption made above that the output price is deterministic.

Entrepreneurs are risk neutral and are protected by limited liability. The distribution of assets in the population of entrepreneurs is given by the cumulative distribution function $G(A)$ on $[0, \infty)$. Investors are risk neutral and demand rate of return equal to 0. Assume that it is optimal to provide the entrepreneurs with incentives to behave.

Varying P, let us compare the level of *aggregate* investment under credit rationing ($B > 0$) and in its absence ($B = 0$), and show that the first- and second-best levels of investment are as depicted in Figure 13.11.

41. See Aghion et al. (2004) for further work on cycles driven by the interest rate conduit.

Further examples of cycles created by credit rationing are investigated in the next section and in Chapter 14.

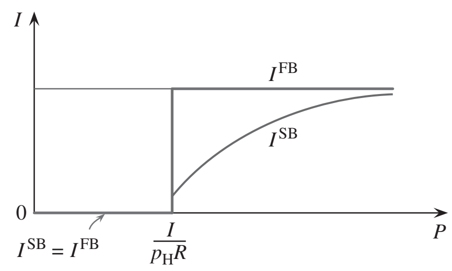


Figure 13.11

Conditional on receiving financing, an entrepreneur's incentive constraint is

$$(\Delta p)R_{\mathrm{b}} \geqslant B,$$

and so the financing condition can be written as $A \geqslant \bar{A}$, where

$$p_{\mathrm{H}}\left[PR - \frac{B}{\Delta p}\right] = I - \bar{A}.$$

The fraction of entrepreneurs who are able to raise funds is

$$X \equiv 1 - G\left(I - p_{\mathrm{H}}\left[PR - \frac{B}{\Delta p}\right]\right), \tag{13.13}$$

or, equivalently, total investment is[42]

$$I^{\mathrm{SB}} = XI,$$

as long as the NPV is positive, i.e., as long as

$$p_{\mathrm{H}}PR - I \geqslant 0.$$

In the positive-NPV range ($P \geqslant I/p_{\mathrm{H}}R$), aggregate investment increases with P as long as $B > 0$. Whether the impact of P on aggregate investment increases with B depends on the derivative of the density. But, as is easily seen in Figure 13.11, *aggregate investment is always more responsive to the output price under credit rationing than with no (or little) credit rationing*, since investment does not move with the output price in this range in the absence of credit rationing.

This illustrates the *net worth effect*: when industry profitability increases, boosting both the pledgeable income and the NPV, and for a given investment level (which is the case here since projects have a fixed size), more and more firms pass the solvency test and get access to financing. This explains the higher responsiveness of investment under financial constraints in the positive-NPV region. As far as investment is concerned, a unit increase in P is tantamount to a uniform increase $p_{\mathrm{H}}R$ in net worths (A).

To complete this analysis, we can now endogenize the product price by assuming the existence of a prior "fraction"[43] X_0 of similar firms that were able to raise financing in the past. The output price is then a decreasing function of total output, $p_{\mathrm{H}}(X_0 + X)R$:[44]

$$P = P(p_{\mathrm{H}}(X_0 + X)R) \quad \text{with } P' < 0. \tag{13.14}$$

An equilibrium is then a "level of investment" XI, where X is obtained from (13.13) and (13.14):

$$X = 1 - G\left(I - p_{\mathrm{H}}\left(P(p_{\mathrm{H}}(X_0 + X)R)R - \frac{B}{\Delta p}\right)\right).$$

Note that as X_0 grows, X must decrease (a crowding-out effect (if X increased, then P would decrease, and so X would decrease after all)) but $X_0 + X$ must increase (there is less than full crowding out (if $X_0 + X$ decreased, then P would increase, and X would increase after all)).

The increased sensitivity of investment under credit rationing, however, is not a general conclusion, as can also be seen from Figure 13.11. A small increase in P from the negative-NPV to the positive-NPV region raises the first-best investment dramatically, but the second-best one less so as not all firms get on board. The effect in force here will become clear in the variable-investment version that we study now.

13.5.1.2 Variable Investment Size

Next consider the variable-investment model of Section 3.4: the number of units of output produced in the case of success is RI, the private benefit in the case of misbehavior BI. To be incentivized to behave, an entrepreneur with investment size I must receive R_{b} in the case of success, such that

$$(\Delta p)R_{\mathrm{b}} \geqslant BI.$$

42. "SB" refers to the "second best," that is, to the situation in which agency costs ($B > 0$) lead to credit rationing. By contrast, "FB" will refer to the "first best," that is, to a situation in which there are no agency costs ($B = 0$) and therefore no credit rationing.

43. If previous-generation entrepreneurs have mass exceeding 1, then X_0 can be greater than 1.

44. We slightly abuse notation by using the same letter P for the price function and its realization.

For a given output price P, an entrepreneur with assets A can borrow up to the level at which pledgeable income is equal to investors' outlay:

$$p_{\mathrm{H}}\left(PR - \frac{B}{\Delta p}\right)I = I - A$$

or

$$I = \frac{A}{1 + (p_{\mathrm{H}}B/\Delta p) - p_{\mathrm{H}}PR}.$$

Assuming, without loss of generality,[45] that all entrepreneurs have the same net worth A, let us analyze the impact of a prior level of investment I_0 on current investment. Total output is then $p_{\mathrm{H}}R(I_0 + I)$. Because

$$P = P(p_{\mathrm{H}}R(I_0 + I)), \quad \text{with } P' < 0,$$

$$I = I^{\mathrm{SB}} = \frac{A}{1 + (p_{\mathrm{H}}B/\Delta p) - p_{\mathrm{H}}P(p_{\mathrm{H}}R(I_0 + I))R}. \tag{13.15}$$

By the same reasoning as in the fixed-investment version, condition (13.15) implies that previous investment partially crowds out current investment:

$$-1 < \frac{\partial I^{\mathrm{SB}}}{\partial I_0} < 0.$$

Let us now compare this sensitivity to that obtained in the absence of credit rationing. In this first-best benchmark, firms maximize their NPV, regardless of their solvency:

$$\max_I \{(p_{\mathrm{H}}PR - 1)I\}.$$

Competitive equilibrium in this constant-returns-to-scale environment implies that unit revenue is equal to unit cost, or

$$p_{\mathrm{H}}P(p_{\mathrm{H}}R(I_0 + I))R = 1. \tag{13.16}$$

Thus, in the absence of credit constraint, past investment fully crowds out current investment:

$$\frac{\partial I}{\partial I_0} = -1.$$

This is due to what might be labeled a *hindering effect of credit rationing*: because part of the benefit from investment expansion accrues to the entrepreneurs and is therefore nonpledgeable, investors are less keen than entrepreneurs to expand as the market becomes more profitable. And so credit rationing may make investment relatively less responsive to market conditions.

Exercise 13.3 pursues this analysis in the intermediate context of variable investment and decreasing returns to scale.

13.5.2 Investment Glut and Dearth Cycles

Let us embed these ideas into a full-fledged dynamic model with overlapping generations of entrepreneurs. The analysis in this section follows that of Suarez and Sussman (1997). The model for each generation is taken to be the constant-returns-to-scale variable-size version.

Generation-t entrepreneurs have mass 1 and are born with net worth A each. They live for two periods, t and $t+1$. The representative generation-t entrepreneur invests I_t at date t. Production occurs, with an output proportional to I_t, at dates t and $t+1$. We make the following assumption.

Assumption 13.2 (time to build). *At date t, only a fraction $\theta < 1$ of investment I_t is operational. The output is θRI_t with probability p_1 and 0 with probability $1 - p_1$. By contrast, the investment becomes fully operational and yields RI_t with probability p_2 and 0 with probability $1 - p_2$ in the second period of its life (that is, at date $t+1$). The investment fully depreciates (is useless) after $t+1$.*

Assumption 13.2 expresses the existence of a time to build if $p_2 \geqslant p_1$ (otherwise, expected output could be greater in the first period of the investment). We therefore assume that, in the absence of moral hazard,

$$p_1 = p_2 = p_{\mathrm{H}}.$$

Let us now introduce moral hazard. Quite generally, a generation-t entrepreneur may misbehave at date t (reduce p_1) and at date $t+1$ (reduce p_2). The reader can follow the steps of the analysis in Section 4.2 to solve for this general case. Because this does not affect the results, we will look at the slightly simpler case of "increasing moral hazard." That is, reflecting the fact that the future is more foreseeable and contractible at short horizons, we assume that moral hazard is more substantial in the second period. Indeed, we assume this in an extreme form: there is no moral hazard in the first period of production, $p_1 = p_{\mathrm{H}}$, and so the date-t income,

45. Recall that with constant returns to scale, all firms are identical up to a scale factor. Put differently, only total net worth matters.

$(p_H \theta R I_t) P_t$, where P_t is the date-t output price, is fully pledgeable to investors. By contrast, date-$(t+1)$ production involves an agency cost: $p_2 \in \{p_L, p_H\}$. The project yields RI_t with probability p_L (the private benefit is then BI_t) or p_H (there is no private benefit). To incentivize the entrepreneur, the latter must receive R_b in case of period-$(t+1)$ success, where

$$(\Delta p) R_b \geqslant BI_t,$$

with $\Delta p \equiv p_H - p_L$.

The financing condition for the generation-t representative entrepreneur is that the pledgeable income exceeds the investors' outlay. If β denotes the discount factor between periods (for investors and entrepreneurs), this condition can be written as

$$\left[p_H \theta R P_t + \beta p_H \left(R P_{t+1} - \frac{B}{\Delta p}\right)\right] I_t \geqslant I_t - A,$$

and so, provided that the NPV per unit of investment is positive, i.e.,

$$(\theta P_t + \beta P_{t+1}) p_H R > 1,$$

the date-t investment is given by

$$\begin{aligned} I_t &\equiv \frac{A}{[1 + \beta p_H B/\Delta p] - [(\theta P_t + \beta P_{t+1}) p_H R]} \\ &\equiv \mathcal{I}(\theta P_t + \beta P_{t+1}), \quad \text{with } \mathcal{I}' > 0. \end{aligned} \tag{13.17}$$

For the sake of comparison, the investment in the absence of credit rationing would maximize the NPV, and so, under constant returns to scale, the unit revenue must be equal to the unit cost in competitive equilibrium:[46]

$$(\theta P_t + \beta P_{t+1}) p_H R = 1. \tag{13.18}$$

In either case (credit rationing or lack thereof), the output price is given by an inverse demand function for the good:[47]

$$P_t = P((\theta I_t + I_{t-1}) p_H R), \quad \text{with } P' < 0. \tag{13.19}$$

The interesting case arises when we make the following assumption.

46. For the moment, we ignore the possibility that investment at date t be equal to 0.

47. Consumers/investors have intertemporal utility

$$\sum_{t \geqslant 0} \beta^t [c_t + \phi(z_t)],$$

where z_t is their consumption of the good in question, c_t is their consumption of numeraire, and ϕ is increasing and concave. Then the inverse demand function is given by $P(z_t) \equiv \phi'(z_t) = P_t$.

Assumption 13.3. $\beta < \theta$.

This assumption states that enough of the investment becomes operational in the first period of its life that the "short-term" price (P_t) matters more than the "long-term" price (P_{t+1}) in the determination of the generation-t investment I_t, whether there is credit rationing ((13.17) holds) or not ((13.18) holds).

Let us show that, under this assumption, the dynamic equilibrium is stationary in the absence of credit rationing, but that it may take the form of an investment (and output) cycle under credit rationing.

13.5.2.1 Absence of Credit Rationing

Let P^* be the stationary price that satisfies the free-entry condition (13.18),

$$(\theta + \beta) P^* p_H R = 1,$$

and let $\hat{P}_t \equiv P_t - P^*$ and $\hat{P}_{t+1} \equiv P_{t+1} - P^*$.

Equation (13.18) yields

$$\hat{P}_{t+1} = -\frac{\theta}{\beta} \hat{P}_t,$$

and, because $\theta/\beta > 1$, a nonstationary price series would diverge. Thus, the only equilibrium with positive investment in each period[48] is a stationary one:

$$P_t = P^* \quad \text{for all } t.$$

13.5.2.2 Credit Rationing

Under credit rationing, investment is given by (13.17). A *two-period cycle*[49] $\{(I^+, P^+), (I^-, P^-)\}$ satisfies

$$I^+ = \mathcal{I}(\theta P^+ + \beta P^-) > I^- = \mathcal{I}(\theta P^- + \beta P^+),$$
$$P^+ = P((\theta I^+ + I^-) p_H R) > P^- = P((\theta I^- + I^+) p_H R).$$

48. There also exists a cycle in which investment occurs every other period. That is, $I_t = I^+$ and $P_t = P^+$ in even periods, say, and $I_t = 0$ and $P_t = P^-$ in odd periods, where

$$(\theta P^+ + \beta P^-) p_H R = 1,$$
$$P^+ \equiv P(\theta I p_H R) > P^- \equiv P(I p_H R).$$

Note that, because $\theta > \beta$,

$$(\theta P^- + \beta P^+) p_H R < 1,$$

and so there is indeed no investment in odd periods.

49. By Sarkovskii's Theorem (see, for example, Theorem 4.3 in Grandmont 1985) cycles of order 2 are in general the "easiest to obtain," then come other cycles with an even period, and finally cycles with an odd-period, three-period cycles being the last to appear.

Such a cycle exists provided that the price and investment functions are "reactive" enough.[50]

13.6 Exercises

Exercise 13.1 (improved governance). There are two dates, $t = 0, 1$, and a continuum of mass 1 of firms. Firms are identical except for the initial wealth A initially owned by their entrepreneur. A is distributed according to continuous cumulative distribution $G(A)$ with density $g(A)$ on $[0, I]$.

Each entrepreneur has a fixed-size project, and must invest I, and therefore borrow $I - A$, at date 0 in order to undertake it. Those entrepreneurs who do not invest themselves, invest their wealth in other firms. The savings function of nonentrepreneurs (consumers) is an increasing function $S(r)$, where r is the interest rate, with $S(r) = 0$ for $r < 0$ (so total savings equal $S(r)$ plus the wealth of unfinanced entrepreneurs). Entrepreneurs have utility $c_0 + c_1$ from consumptions c_0 and c_1.

A project, if financed, yields $R > 0$ at date 1 with probability p and 0 with probability $1 - p$. The probability of success is $p_{\rm H}$ if the entrepreneur works and $p_{\rm L} = p_{\rm H} - \Delta p$ if she shirks. The entrepreneur obtains private benefit B by shirking and 0 otherwise. Assume $p_{\rm H}R > I > p_{\rm H}(R - B/\Delta p)$, that financing cannot occur if the entrepreneur is provided with incentives to misbehave, and that the equilibrium interest rate is strictly positive.

(i) What is the pledgeable income? Write the financing condition.

(ii) Give the expression determining the market rate of interest. How does this interest rate change when improved investor protection lowers B?

Exercise 13.2 (dynamics of income inequality). (This exercise builds on the analysis of Section 13.4 and on Matsuyama (2000).)

(i) Consider the "warm-glow" model: generations are indexed by $t = 0, 1, \dots, \infty$. Each generation lives for one period; each individual has exactly one heir. A generation-t individual has utility from consumption c_t and bequest $\mathcal{L}_t$ equal to

$$\left(\frac{c_t}{1-a}\right)^{1-a}\left(\frac{\mathcal{L}_t}{a}\right)^{a}$$

with $0 < a < 1$.

What is the individual's utility from income y_t?

(ii) Consider the entrepreneurship model of Section 13.4, with two twists:

- variable-size investment (instead of a fixed-size one),
- intraperiod rate of interest r (so investors demand $(1 + r)$ times their outlay, in expectation); r is assumed constant for simplicity.

One will assume that $p_{\rm H} = 1$ and that each generation t is born with endowment $\hat{A}$ (to which is added bequest $\mathcal{L}_{t-1}$, so $A_t = \hat{A} + \mathcal{L}_{t-1}$). See Figure 13.12.

A successful project delivers $RI \geqslant (1 + r)I$, an unsuccessful one 0. The private benefit from misbehaving, BI, is also proportional to investment.

Let

$$\rho_1 \equiv R \quad \text{and} \quad \rho_0 \equiv R - \frac{B}{\Delta p}.$$

Assume that

$$a(\rho_1 - \rho_0) < 1 - \frac{\rho_0}{1 + r}.$$

Show that each dynasty's long-term wealth converges to

$$A_\infty \equiv \frac{\hat{A}}{1 - a(\rho_1 - \rho_0)/(1 - \rho_0/(1 + r))},$$

regardless of its initial total wealth A_0 (that is, $\hat{A}$ plus the bequest from generation -1, if any).

(iii) Now assume that there is a minimal investment scale $\underline{I} > 0$ below which nothing can be produced. For $I \geqslant \underline{I}$, the technology is as above (constant returns to scale, profit RI in the case of success, private benefit BI in the case of misbehavior, etc.).

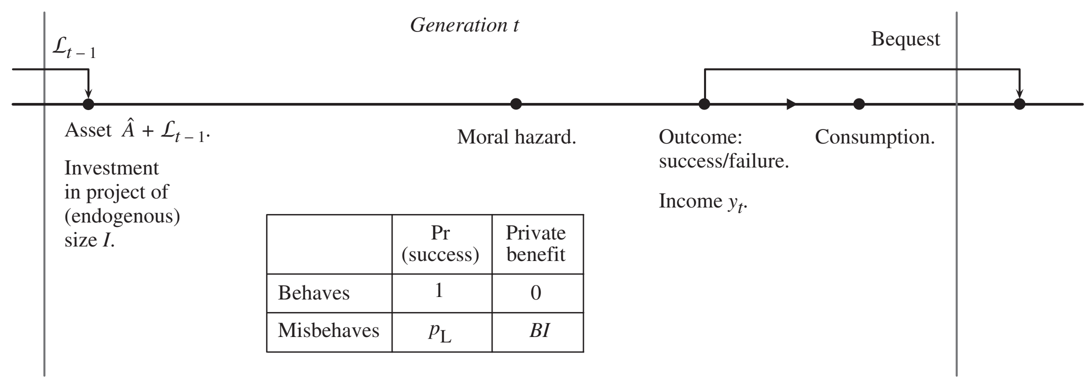
Compute the threshold A_0^* under which the dynasty remains one of lenders (at rate r) and never makes it to entrepreneurship.

What is the limit wealth $A_\infty^{\rm L}$ of these poor dynasties? (The limit wealth of dynasties starting with $A_0 \geqslant A_0^*$ is still A_∞.)

50. Let $\hat{P}^+ \equiv P^+ - P^{**}$ and $\hat{P}^- \equiv P^- - P^{**}$, where P^{**} is the steady-state price corresponding to equations (13.17) and (13.19). The local mapping from, say, $\hat{P}^+$ into itself around $\hat{P}^+ = 0$ has slope

$$\left[\frac{P'\mathcal{I}'(p_{\rm H}R)(\theta(1+\beta))}{1 - P'\mathcal{I}'(p_{\rm H}R)(\theta^2+\beta)}\right]^2.$$

Because $P'\mathcal{I}' < 0$ and $\theta^2 + \beta < \theta(1 + \beta)$ from Assumption 13.3, this slope is greater than 1 provided $P'\mathcal{I}'$ is sufficiently large at P^{**}.



	Pr (success)	Private benefit
Behaves	1	0
Misbehaves	p_L	BI

Figure 13.12

(iv) Finally, close the model by assuming that investors are domestic investors and by describing the equilibrium in the loan market. Focus on *steady states*. Show that multiple steady states may coexist:

- one in which everyone (investors, entrepreneurs) has the same wealth and

$$\rho_1 = 1 + r,$$

- others, with unequal wealth distribution, in which $\rho_1 > 1 + r$, a fraction κ of the population is poor (lends), and a fraction $1 - \kappa$ is rich (borrows to undertake projects).

Exercise 13.3 (impact of market conditions with and without credit rationing). This analysis pursues that of Section 13.5.1. There, we compared the sensitivity of investment with the output price (or installed-base investment) in the presence or absence of credit rationing, focusing on either the fixed-investment variant or the constant-returns-to-scale variant. We now assume decreasing returns to scale.

The representative entrepreneur (there is a unit mass of such entrepreneurs) has initial wealth A, is risk neutral and protected by limited liability, and invests $I + K$, where I is the scale of investment and K a fixed cost that is unrelated to scale. We assume that $K \geqslant A$, and so investors are unable to finance by themselves even a small investment.

An entrepreneur is successful with probability p and fails with probability $1 - p$. We assume that the shocks faced by the entrepreneurs are independent. This hypothesis is consistent with the assumption made below that the output price is deterministic. When successful, the entrepreneur produces $R(I)$ units of a good (with $R(0) = 0$, $R' > 0$, $R'' < 0$, $R'(0) = \infty$, $R'(\infty) = 0$); an entrepreneur who fails produces nothing. For concreteness, let

$$R(I) = I^{\alpha}, \quad \text{with } 0 < \alpha < 1.$$

As usual, the probability of success is endogenous: $p \in \{p_L, p_H\}$. Misbehavior, $p = p_L$ (respectively, good behavior, $p = p_H$), brings about private benefit BI (respectively, no private benefit). To prevent moral hazard, the entrepreneur must receive reward R_b in the case of success, such that

$$(\Delta p) R_b \geqslant BI.$$

The product sells at price P per unit. Presumably, investors are risk neutral and demand rate of return 0.

Suppose that the fixed cost K is "not too large" (so that the entrepreneur wants to invest in the absence of credit rationing), and that

$$\frac{p_H B}{\Delta p} < \frac{1 - \alpha}{\alpha}.$$

(i) Derive the first- and second-best investment levels as functions of P. Show that they coincide for $P \geqslant P_0$ for some P_0.

(ii) Using a diagram, argue that there exists a region of output prices in which the second-best investment is more responsive than the first-best investment to the output price.

(iii) How would you analyze the impact of the existence of an installed-base level of investment I_0?

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